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THE EFFECT OF TRANSVERSE VIBRATION ON FREE CONVECTION FROM A HORIZONTAL CYLINDER

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Abstract—An electrically heated, horizontal, 0·049 inch diameter cylinder in free convection to water and aqueous glycerine was vibrated vertically at frequencies ranging from 17 to 37 c/s at displacement amplitudes of up to 0·086 in. The coefficient of heat transfer was found to increase with frequency and amplitude, with some increases exceeding ten-fold. A dimensionless correlation was devised for the present results together with those of three other investigations.

NOMENCLATURE

a, displacement amplitude;
b, constant in equation (1);
D, diameter of cylinder;
D_v, diffusivity;
f, frequency;
g, acceleration of gravity;
Gr, Grashof number, $D^3g\beta(t_w - t_l)/\nu^2$ for heat transfer and $D^3g\Delta\rho/\rho\nu^2$ for mass transfer;
h, coefficient of heat transfer;
h', coefficient of heat transfer without vibration but in the same fluid and for the same diameter and same Pr and Gr as with vibration;
k_G, coefficient of mass transfer;
k_G', coefficient of mass transfer without vibration but for the same independent non-vibratory variables as with vibration;
m_1, m_2, m_3, m_4, exponential constants in equation (1);
Pr, Prandtl number, ν/α;
Re_v, vibrational Reynolds number, $D(4af)/\nu$;
Sc, Schmidt number, ν/D_v;
t_l, bulk temperature of fluid;
t_w, surface temperature of cylinder.

Greek symbols
α, thermal diffusivity;
β, thermal coefficient of volumetric expansion;
ϕ, function represented by the curve of Fig. 2;
$\Delta\rho$, fluid density difference between surface and bulk;
ρ, fluid density;
ν, kinematic viscosity.

INTRODUCTION

THE EFFECT of vibration on heat transfer and mass transfer has been a subject of interest to various investigators [1]. Experimental studies have been reported for forced convection and to a lesser extent for free convection. For transversely vibrating horizontal cylinders in free convective heat transfer, Martinelli and Boelter [2], Boelter and Mason [3], and Deaver *et al.* [4] have worked in water, and Lemlich [5] and Fand and Kaye [6] have worked in air. Lemlich employed vertical and horizontal transverse vibration but found no significant difference between their effects. In addition, Lemlich and Levy [7] reported for free convective mass transfer from naphthalene and D-camphor to air.

All in all, there are only a few experimental studies available for transversely vibrating horizontal cylinders in free convection. Furthermore, the two earliest studies with water [2, 3], although closely related to each other, reported results which are not in mutual agreement. Accordingly, the present investigation was embarked upon to measure and correlate the effect of transverse vibration on free convective heat transfer from a horizontal cylinder to liquid water and aqueous glycerine.

EXPERIMENT

The apparatus consisted essentially of equipment and instruments to stretch an electrically heated length of 0·049 in O.D. stainless steel (hypodermic) tubing horizontally through a glass tank containing liquid, vibrate the tube at various amplitudes and frequencies, and measure the pertinent variables involved. A schematic diagram is shown in Fig. 1.

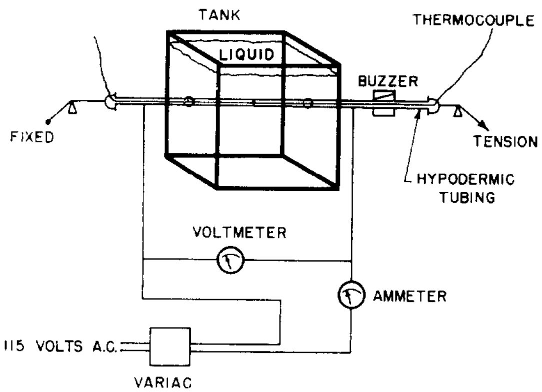

FIG. 1. Apparatus.

The inner dimensions of the tank were 18 × 18 × 24 in. The tube passed lengthwise through the tank via two 0·5 in diameter holes in the opposite square faces of the tank. A liquid seal was maintained at each hole by means of very thin rubber sheeting cemented to the glass wall with an epoxy resin and held around the tube with rubber bands.

The tube was stretched taut by means of auxiliary wires attached at each end and pulled over bridges. Vertical vibration was induced by a flick of the finger, and then maintained at resonance through contact with the armature of an electric buzzer. The frequency of vibration was varied by changing the tension on the tube. The amplitude of vibration was varied by changing the voltage to the buzzer or by adjusting the position of the armature.

Frequency was measured with a calibrated stroboscopic light. It ranged from 17 to 37 c/s. Displacement amplitude was measured with a calibrated travelling telescope mounted on a scale. It ranged from 0·023 to 0·086 in with distilled water, and from 0·038 to 0·070 in with 27·3 per cent glycerine in water solution.

The tube was heated by passing an electric current along its submerged length. Heat dissipation was measured electrically. Bulk liquid temperature was measured thermometrically.

The temperature of the tubing was sensed internally with a calibrated silver-soldered chromel–alumel thermocouple sheathed in polyethylene which served as electrical insulation along its entire length. This assembly was threaded through the tube until the thermocouple junction was at the center of the tank. (This was also the location at which the amplitude was measured.)

Placing the thermocouple internally avoided any spurious effects that external placement on the outer surface might have had. Such effects could have been serious here in view of the relatively high coefficients of heat transfer obtainable in liquids.

The thermal resistance of the metal tube wall was quite small, even compared to that of the boundary layer. Accordingly, the temperature drop across the metal wall was neglected and the internal temperature was taken as the surface temperature. Of course, the thermal resistance of the polyethylene sheath was of no consequence since all the heat was generated externally to it, and it contained no internal steady-state heat sink.

The temperature difference between the tube and the bulk liquid ranged between 7° and 58°F without vibration, and between 9° and 28°F with vibration. The bulk temperature itself ranged from 83° to 93°F.

RESULTS

Tabulated results are summarized in Table 1. Included are a total of 25 control runs, conducted without vibration, to serve as a standard of comparison. Results for Nusselt number from these control runs average about 10 per cent higher than McAdams' generalized correlation for free convection from horizontal cylinders [8].

Additional control runs (not tabulated) were conducted with vibration but with no electrical heating. These showed no significant tube temperature rise accompanying the vibration. This in turn indicates that no appreciable heat was generated by drag.

Table 1. *Tabulated results*

Run	Liquid	Tube temp., t_w (°F)	Bulk temp., t_l (°F)	Frequency (c/s)	Displacement amplitude (ft)	Heat-transfer coefficient (Btu/h ft^2 degF)	h/h'
1	Water	93·7	84·5	0	0	312	—
2	Water	98·5	84·6	0	0	247	—
3	Water	106·7	84·2	0	0	312	—
4	Water	104·9	84·2	0	0	303	—
5	Water	111·5	84·2	0	0	316	—
6	Water	116·9	84·2	0	0	311	—
7	Water	114·2	83·3	0	0	308	—
8	Water	111·5	83·3	0	0	300	—
9	Water	107·0	83·3	0	0	280	—
10	Water	126·7	83·3	0	0	334	—
11	Water	120·7	83·3	0	0	317	—
12	Water	117·8	83·3	0	0	313	—
13	Water	119·9	83·3	0	0	315	—
14	Water	129·5	83·3	0	0	333	—
15	Aq. glycerine	103·5	83·5	0	0	203	—
16	Aq. glycerine	115·8	83·5	0	0	248	—
17	Aq. glycerine	120·6	83·5	0	0	253	—
18	Aq. glycerine	109·8	83·5	0	0	217	—
19	Aq. glycerine	125·1	83·7	0	0	283	—
20	Aq. glycerine	129·9	83·7	0	0	281	—
21	Aq. glycerine	133·9	83·7	0	0	289	—
22	Aq. glycerine	138·8	83·7	0	0	288	—
23	Aq. glycerine	142·3	83·7	0	0	303	—
24	Aq. glycerine	135·6	83·7	0	0	304	—
25	Aq. glycerine	122·0	83·7	0	0	252	—
26	Water	97·4	88·1	36·7	0·00715	3120	12·28
27	Water	104·0	88·8	36·0	0·00599	2710	9·82
28	Water	103·5	89·0	35·4	0·00517	2410	8·80
29	Water	105·4	89·6	17·0	0·00615	2000	7·21
30	Water	104·0	90·0	35·5	0·00648	2940	10·80
31	Water	110·9	90·8	34·3	0·00451	2670	9·20
32	Water	114·7	90·4	35·0	0·00484	2570	8·58
33	Water	110·3	91·8	33·8	0·00336	2210	7·71
34	Water	120·2	92·0	33·8	0·00304	2330	7·55
35	Water	119·9	92·4	35·1	0·00533	2460	8·35
36	Water	103·3	88·1	19·2	0·00664	1830	6·62
37	Water	99·9	89·0	35·8	0·00484	2730	10·45
38	Water	108·1	89·5	35·2	0·00285	2250	7·84
39	Water	104·2	90·0	35·4	0·00320	2480	9·06
40	Water	113·5	90·0	34·0	0·00369	1970	6·61
41	Water	102·1	90·3	34·2	0·00320	2300	8·69
42	Water	99·9	91·0	34·0	0·00465	3050	12·11
43	Water	111·8	91·2	33·8	0·00352	1910	6·53
44	Water	99·9	91·8	32·8	0·00320	2190	8·83
45	Water	99·9	92·0	33·2	0·00484	3080	12·44
46	Water	108·1	92·0	33·0	0·00468	2380	8·51
47	Water	113·1	92·7	30·3	0·00582	2380	8·16
48	Water	117·5	92·7	29·3	0·00386	1970	6·54
49	Water	105·1	93·2	35·0	0·00188	1790	6·71
50	Aq. glycerine	97·6	84·0	34·4	0·00335	1590	9·35
51	Aq. glycerine	100·8	84·0	35·0	0·00450	1680	9·15
52	Aq. glycerine	105·6	84·0	34·8	0·00485	1560	7·72
53	Aq. glycerine	106·9	84·0	34·8	0·00468	1560	7·55

Table 1—continued

Run	Liquid	Tube temp., t_w (°F)	Bulk temp., t_l (°F)	Frequency (c/s)	Displacement amplitude (ft)	Heat-transfer coefficient (Btu/h ft² degF)	h/h'
54	Aq. glycerine	106·4	84·0	35·2	0·00500	1600	7·79
55	Aq. glycerine	111·8	84·1	35·3	0·00500	1550	6·93
56	Aq. glycerine	111·7	84·1	35·7	0·00485	1560	6·96
57	Aq. glycerine	114·0	85·7	35·5	0·00485	1650	7·28
58	Aq. glycerine	100·8	86·0	34·8	0·00335	1130	6·43
59	Aq. glycerine	105·6	87·8	34·8	0·00435	1450	7·67
60	Aq. glycerine	109·1	87·8	34·8	0·00320	1460	7·26
61	Aq. glycerine	107·2	87·8	35·2	0·00385	1600	8·22
62	Aq. glycerine	111·3	87·9	35·0	0 00385	1500	7·14
63	Aq. glycerine	109·7	88·0	35·7	0 00468	1620	7·96
64	Aq. glycerine	112·2	88·1	35·5	0 00450	1610	7·60
65	Aq. glycerine	115·2	88·4	35·3	0·00450	1650	7·43
66	Aq. glycerine	102·9	88·9	36·1	0·00517	1870	10·94
67	Aq. glycerine	104·7	89·1	35·7	0·00485	2060	11·48
68	Aq. glycerine	108·6	89·5	35·6	0·00450	2000	10·37
69	Aq. glycerine	111·2	90·0	35·6	0·00485	2060	10·22
70	Aq. glycerine	116·2	90·0	35·6	0·00468	1920	8·75
71	Aq. glycerine	107·2	86·0	17·4	0·00582	1030	5·10
72	Aq. glycerine	110·7	86·0	17·0	0·00435	1000	4·66

A total of 47 runs were carried out with vibration applied to the electrically heated tube. Here too, the tube is essentially a horizontal cylinder despite the variation in amplitude along its bowed length. This is because even the largest amplitude employed is very small compared to the total length of the tube. In addition, it can be shown that longitudinal conduction along the tube is negligible [5].

Fluid properties were taken at the arithmetic average film temperature. *Pr* for water ranged from approximately 4 to 5, and for the aqueous glycerine from approximately 10 to 12.

The last column of Table 1 lists the ratio of the heat-transfer coefficient with vibration to that without vibration. The denominator h' was taken from smoothed results of control runs without vibration in the same liquid at the same $t_w - t_l$ as with vibration. Thus *Pr* and *Gr* for h' are substantially the same as for *h* since the bulk temperature did not vary much.

It is evident from this listing of h/h' that a considerable increase in coefficient was obtained with vibration. In several instances this increase exceeded ten-fold.

The general trend of results confirms that of earlier studies. The ratio h/h' generally increased with amplitude and with frequency.

CORRELATION

A single dimensionless correlation was devised for the present results and the results of several previous investigations. This was accomplished through the following steps:

1. The independent variables were selected as the more or less usual dimensionless groups (5) Re_v, a/D, Gr, and Pr. The dependent variable was taken as $(h/h') - 1$, which is the fractional increase in coefficient. This particular choice for the dependent variable has certain advantages [5], among them the partial cancellation of experimental error by virtue of the ratio.
2. A relationship of the form of equation (1) was hypothesized.

$$\frac{h}{h'} - 1 = b\,(Re_v)^{m_1} \left(\frac{a}{D}\right)^{m_2} (Gr)^{m_3}\,(Pr)^{m_4} \tag{1}$$

3. Multiple linear regression analysis was applied with the logarithm of equation (1)

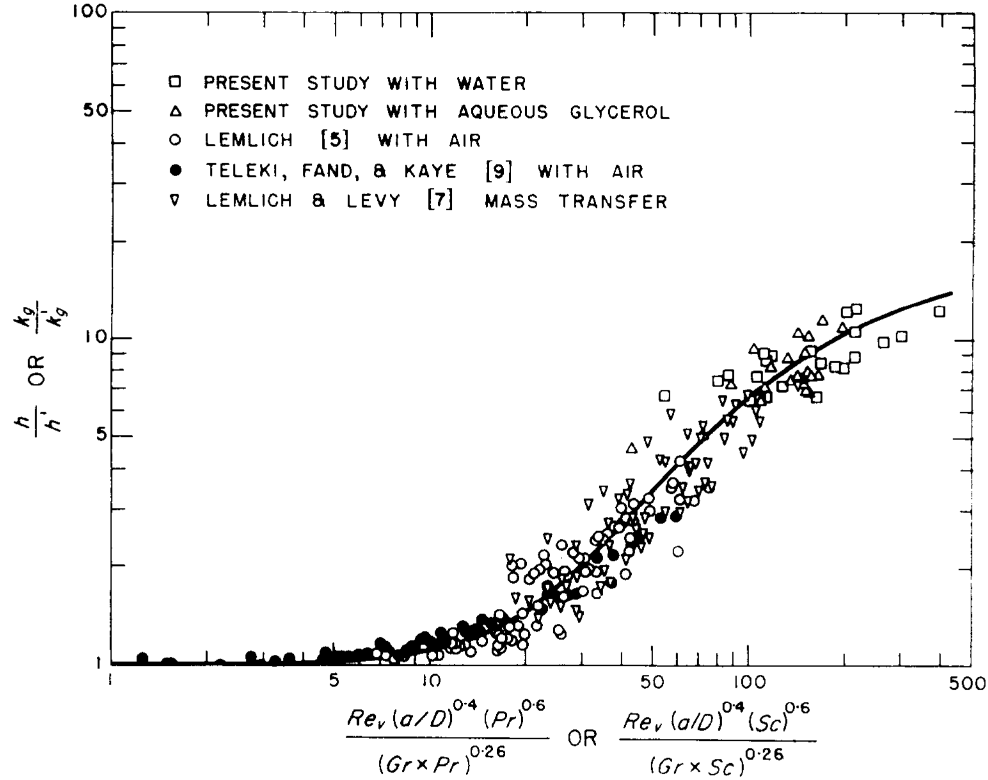

FIG. 2. Correlation.

via an IBM 1620 digital computer. This yielded a fair fit. However, the results upon logarithmic plotting were slightly curved—indicating the assumption of linearity was not quite correct.

4. Therefore, using the exponents obtained in step 3 as an initial guide, trial alterations were made in these exponents until the best appearing fit was obtained for the curve.
5. The results were then replotted on logarithmic coordinates with h/h' as the ordinate rather than $(h/h') - 1$ since the former approaches a finite low asymptote (viz. unity) on these coordinates while the latter does not.

This final plot appears as an S-curve, and is shown in Fig. 2. The results employed in the correlation include those of the present study plus three others [5, 6,* 7]. For these three others taken together, cylinder diameter ranges from 0·0253 to 0·875 in, frequency ranges from 20 to 225 c/s, and displacement amplitude ranges up to 0·16 in.

One of these three investigations is for mass transfer [7]. Its use here requires interchange of analogous dimensionless groups, viz. k_G/k_G' for h/h', Sc for Pr, and mass transfer Gr for heat transfer Gr.

It was deemed safest to omit the two mutually disagreeing investigations [2, 3] mentioned earlier.

The third investigation with water [4] involved much lower frequencies (0 to 4·25 c/s) and much higher a/D (up to 394) than those for the four investigations employed in the correlation. Values of h/h' for this third investigation did

* Note: The data of the original report [9] were actually used.

not correlate well with results for the four investigations. Accordingly, this third investigation was not included in the correlation.*

The correlation states that

$$\frac{h}{h'} = \phi\left[\frac{Re_v\,(a/D)^{0\cdot4}\,Pr^{0\cdot6}}{(Gr \times Pr)^{0\cdot26}}\right] \qquad (2)$$

where ϕ is given by the curve of Fig. 2 (and where Pr is replaced by Sc for mass transfer). The grouping of the independent variables is revealing. The numerator $Re_v(a/D)^{0\cdot4}Pr^{0\cdot6}$ represents the vibrational disturbance in the form of a modified forced convection term. The exponent on the Prandtl number, which was found to be 0·6, accords well with ordinary forced convective crossflow for which case the exponent on the Prandtl number is also about 0·6 of that on the Reynolds number.

The denominator involves only the product $Gr \times Pr$. This product, of course, is characteristic of ordinary free convection. Thus the overall grouping of independent variables shows a forced convective numerator, accounting for vibration, which competes for relative effect (i.e. effect on h/h') with a free convective denominator. So when ordinary free convection is strong, the relative effect of a given vibrational disturbance is weakened, and when ordinary free convection is weak the relative effect of vibration is strengthened. (It must be emphasized that it is the effect on h/h' which is referred to here, and not the effect on h.)

For vibrational disturbances which are too weak in comparison with ordinary free convection, no significant increase in coefficient is to be expected. This is confirmed by Fig. 2 which shows that below an abscissa of roughly 10 the increase in coefficient is quite small.

SUMMARY OF CONCLUSIONS

Vertically vibrating a cylinder in free convection to water or aqueous glycerine markedly increases the heat-transfer coefficient, some increases exceeding ten-fold having been observed. Raising the frequency or the amplitude increases the effect.

Results of four investigations, including the present one, are correlated by Fig. 2. The form of the correlation supports the notion that the increase in coefficient depends on the relative strength of the vibrational disturbance compared to that of ordinary free convection.

* Note: The correlation proved inapplicable to some results reported [10] for the "inverse" system in which the cylinder is stationary and the medium vibrates horizontally (at sonic frequencies).

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REFERENCES

1. R. LEMLICH, Vibration and pulsation boost heat transfer, *Chem. Engng* **68**, 171–176 (1961).
2. R. C. MARTINELLI and L. M. K. BOELTER, The effect of vibration on heat transfer by free convection from a horizontal cylinder, *Proc. 5th Int. Cong. Appl. Mech.* p. 578 (1938).
3. L. M. K. BOELTER and W. E. MASON, Vibration—its effect on heat transfer, *Power Plant Engng* **44**, 43–46 (1940).
4. F. K. DEAVER, W. R. PENNEY and T. B. JEFFERSON, Heat transfer from an oscillating horizontal wire to water, *Trans. Amer. Soc. Mech. Engrs, Series C, J. Heat Transfer* **84**, 251–256 (1962); Also personal communication from T. B. JEFFERSON (1961).
5. R. LEMLICH, Effect of vibration on natural convective heat transfer, *Industr. Engng Chem.* **47**, 1175–1180 (1955), and correction, **53**, 314 (1961). Supporting data filed in Ph.D. Dissertation, University of Cincinnati, Cincinnati, Ohio (1954).
6. R. M. FAND and J. KAYE, The influence of vertical vibrations on heat transfer by free convection from a horizontal cylinder, *International Developments in Heat Transfer*, pp. 490–498. ASME (1961).
7. R. LEMLICH and M. R. LEVY, Effect of vibration on natural convective mass transfer, *J. Amer. Inst. Chem. Engrs* **7**, 240–242 (1961). Supporting data filed in Chemical Engineering Department, University of Cincinnati, Cincinnati, Ohio.
8. W. H. MCADAMS, *Heat Transmission*, p. 176. McGraw-Hill, New York (1954).
9. C. TELEKI, R. M. FAND and J. KAYE, Influence of vertical vibrations on the rate of heat transfer from a horizontal cylinder in air, Wright Air Development Command, *TN* 59-357 (1960).
10. R. M. FAND and J. KAYE, Effect of high intensity stationary and progressive sound fields on free convection from a horizontal cylinder, *TN* 59-18 (1959).

Résumé—Un cylindre horizontal de 1,2 mm de diamètre, chauffé électriquement et refroidi par convection naturelle dans de l'eau ou une solution aqueuse de glycérine, a été mis en vibration verticalement à des fréquences allant de 17 à 37 Hertz et à des amplitudes allant jusqu'à 2,2 mm. On a trouvé que le coefficient de transport augmentait avec la fréquence et l'amplitude, multiplié parfois plus de dix fois. Une corrélation sans dimensions a été imaginée pour les résultats actuels en même temps que pour ceux de trois autres études.

Zusammenfassung—Ein elektrisch geheizter, bei freier Konvektion horizontal in Wasser bzw. in mit Wasser verdünntem Glyzerin liegender Zylinder von 1,25 mm Durchmesser wird senkrecht zu seiner Lage in Schwingung versetzt mit Frequenzen von 17 bis 37 Hertz bei Amplituden von einer Auslenkung bis zu 2,18 mm. Es ergab sich, dass die Wärmeübergangszahl mit der Frequenz und der Amplitude grösser wurde, wobei manche Steigerungen das 10-fache überschritten. Für die vorliegenden Resultate und gleichzeitig für die Ergebnisse von drei anderen Forschungsarbeiten wurde eine Beziehung mit dimensionslosen Grössen aufgestellt.

Аннотация—Рассматриваются вертикальные колебания подогреваемого электрическим. током горизонтального цилиндра диаметром 1,245 мм при наличии свободной конвекции в воде и жидком глицерние. Частота колебаний цилиндра от 17 до 37 гц, а амплитуда смещения до 2,234 мм. Установлено возрастание (в некоторых случаях более чем в 10 раз) коэффициента теплообмена с увеличением частоты и амплитуды. Результаты данного и трех других исследований обобщены в безразмерном виде.